Bound states of massless fermions as a source for new physics

Opening lecture given at the 33rd course of the International School of Subnuclear Physics, Erice, Sicily (1995)

V. N. Gribov [a,1]

[a] *Landau Institute for Theoretical Physics, ul. Kosygina Moscow, Russia*
and
KFKI Research Institute for Particle and Nuclear Physics, H-1525 Budapest 114, P.O.B. 49, Hungary

Abstract

The contribution of interactions at short and large distances to particle masses is discussed in the framework of the standard model.

1 Introduction

In this opening lecture I would like to discuss the standard model from a point of view different from the usual discussion of this theory.

What is the Weinberg-Salam-Glashow standard model ? We have three interactions: $SU(3)$, $SU(2)$ and $U(1)$ and three generations of quarks and leptons. All particles in this theory are supposed to be intrinsically massless. In order to connect this theory with the real world, the so-called Higgs sector is introduced, which is responsible for fermionic masses and the masses of W^+, W^- and Z^0.

While the first part of the theory feels beautiful, the second part of it looks ugly to almost everybody. What is the reason for that ? The first part - massless gauge bosons and massless fermions - has several remarkable properties. At the first sight it contains no dimensional parameters, only dimensionless couplings g_1, g_2, g_3. But, as it was discovered in 1957 by Landau, Abrikosov and Khalatnikov [1], the Abelian coupling is increasing with momentum, while

[1] E-mail: gribov@itkp.uni-bonn.de

the non-Abelian couplings g_2, g_3, as found by Gross, Politzer and Wilczek [2], are decreasing with it. Experimentally we know, that $\alpha_3 = \frac{g_3^2}{4\pi}$ is of the order of unity in the momentum region $\lambda_3 \sim 1$ GeV and $\alpha_1 = \frac{g_1^2}{4\pi}$ is of the order of unity at the scale $\lambda_L = 10^{38}$ GeV; g_2 is small everywhere. As a result, instead of dimensionless couplings we have here two scales, short and large distance ones different by 10^{38} orders of magnitude. Between these two scales all interactions are weak.

Introducing the Higgs sector, we are introducing a new scale due to the running of the Higgs self-coupling λ - a scale, where this λ is of the order of unity. If the Higgs boson was light enough, this new scale would be relatively large. However, no light Higgs boson was found so far and in this sense the theory of the trivial Higgs sector is close to being in trouble.

There were many proposals - supersymmetry, grand unification, supergravity etc. - which corresponded to the introduction of new interactions between the two scales I mentioned. Some of them may be correct - we don't know. But to my knowledge nobody was able yet to construct a theory which is asymptotically completely free in the sense that it does not contain strong interactions at short distances like $U(1)$ interactions.

The aim of my lecture is to raise the question, whether it is really necessary to introduce new interactions between those two scales in order to explain the nature, and to present some observations which indicate that this might not be necessary.

We believe, that strong $SU(3)$ interactions are able to prevent quarks from being observed as real particles and to organize bound states which we call hadrons. Why then the strong $U(1)$ interaction at the scale $q^2 \sim \lambda_L^2$ is unable to give masses to quarks and leptons and their bound states which would be the Higgs boson and the longitudinal components of W^+, W^- and Z^0 ? It is important to understand that the two phenomena - the existence of bound states and the generation of masses - are very strongly connected. If the interaction is strong enough to create massless bound states, these bound states will condensate and the scattering of massless fermions on this condensate will result in the propagation of fermions as massive objects. In this sense the source for the mass generation is the existence of massless bound states of massless fermions. What kind of bound states can we expect ? If we have a doublet of left-handed quarks interacting with right-handed antiquarks through the $U(1)$ exchange, they could create a massless spinless doublet. Together with an anti-doublet it will give us four states which, eventually, become massive bosons - the Higgs boson and three longitudinal components of W^+, W^- and Z^0. The last step occurs at large distances: between the Compton wave length of the quark and λ_L^{-1}. In order to understand how this can happen, let us discuss the space-time structure of these "intrinsically" massless bosons. Due to the fact,

that these bound states are created by the strong interactions on the distances 10^{-52} cm, one might expect that they will be point-like. However, although there is very little knowledge about the structure of relativistic bound states, these expectations do not seem to be correct. Even in the non-relativistic quantum mechanics this expectation is not true in general, since the size of a bound state is determined by the binding energy and not by the radius of the forces. A famous example for this is the deuteron in the limit when the radius of nuclear forces is going to zero. In this limit the proton and the neutron inside the deuteron are always outside the region where the interaction is different from zero. In reality, in addition to nuclear forces there exist always long-range electromagnetic interactions between the proton and the neutron. The binding energy depends on both interactions and would not be zero even if the binding energy due to solely nuclear interactions were exactly zero. This is just an analogue of the mechanism by which a massless boson acquires a mass. In the latter case the role of e.m. interactions between the proton and the neutron is played by the interaction with the condensate and with the colour field. The size of the new massive state would be determined by the masses of the constituent fermions.

These considerations suggest, that the boson masses could be calculated as functions of fermionic masses, with little sensitivity to the lack of knowledge about the interaction structure at the distances of the order of $1/\lambda_L$. In the next section I will present these calculations. They contain a factor $\ln \frac{\lambda_L^2}{\lambda_3^2}$ and give reasonable predictions for masses of the top quark and the Higgs meson. In the third section of this lecture we shall discuss another topics - the π meson mass. The π meson is a bound state of a light q and a light $\overline{q}$ interacting via the exchange of coloured gluons, which becomes a massless Goldstone in the limit when the masses of light quarks tend to zero. But the light quark participates in both $U(1)$ and $SU(3)$ interactions and thus plays a role in the creation not only of pions but also of massless Goldstones which are the source of the longitudinal components of W and Z. This means, that these two interactions, the long range $SU(3)$ and the short range $U(1)$ produce two types of zero mass bound states. It is natural to expect, that these states will strongly influence each other. It is possible to calculate the interaction between these two states. As we will show, this gives an expression for the π meson mass as an integral over light quark masses and this integral contains the same $\ln \frac{\lambda_L^2}{\lambda_3^2}$ as the Higgs mass. Knowing the π meson mass and using the same value for the Landau scale, we find a reasonable value for the constituent light quark mass.

The conclusion of sections 2 and 3 is, that measuring masses of some bound states of quarks and leptons we obtain information about the scale where the fermionic masses are created, and this information is roughly in agreement with the idea that masses are generated at the Landau scale. If proved to be true, this would suggest, that the other interactions, even if they exist between

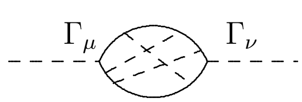
the two scal

es, are not important in the process of creation of fermion masses.

Before going to these rather simple calculations, let us discuss, what kind of problems will arise if one assumes that $U(1)$ interactions are responsible for the fermionic masses. The source of the problem is obvious: there are many quarks and leptons, so why do we expect to have only four intrinsically massless bosons ? Indeed, these bosons are proved to be complicated superpositions of different quarks and leptons. Where are the other massless states which we have to expect on the basis of the $U(1)$ symmetry ? A possible answer is the following. The interaction I call $U(1)$ is well defined only in the region of momenta much smaller than λ_L; in the region $q^2 \geq \lambda_L^2$ where the coupling is large we are not able to write any Lagrangians, since in this region the $U(1)$ interaction could be considered as a very complicated fermionic interaction or even an interaction which contains an additional horizontal gauge field connecting different generations. The only condition imposed on this interaction is, that it has to have no logarithmic tails into the $q^2 \ll \lambda_L^2$ region. It is not clear at all, whether the interaction is uniquely defined by this condition. The strongest objection to a theory of this type is usually connected with the existence of a large number of non-anomalous currents which are conserved for massless fermions; for any of these currents there have to be corresponding Goldstones. This argument is, however, not so serious as it looks. The contribution of the Goldstones to the currents is proportional to the product $fg = m$ where f is the amplitude for the Goldstone current transition, g is the analogue of the Yukawa coupling and m is the fermion mass. If f is of the order of λ_L, then $g \sim \frac{m}{\lambda_L}$ and the Goldstone will not interact with fermions.

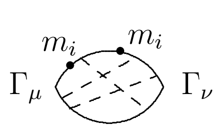
2 Masses of vector bosons and Higgs bosons

Let us consider the polarization operator for $SU(2)$ vector bosons. It can be written as

$\Pi_{\mu\nu}(k) =$ 

(1)

with vertices in $\Pi_{\mu\nu}$ of the form $\frac{1}{2}\tau_\alpha \frac{1}{2}(1-\gamma_5)\gamma_\mu$. For massless fermions $\Pi_{\mu\nu}$ has the form $\Pi_{\mu\nu} = \Pi^{\perp}_{\mu\nu} = (\delta_{\mu\nu}k^2 - k_\mu k_\nu)\,\Pi(k^2)$, and it corresponds to the massless W. Fermionic masses lead to an additional term $\Pi'_{\mu\nu}$ in $\Pi_{\mu\nu}$:

$\Pi'_{\mu\nu} =$ 

$$= \delta_{\mu\nu} \frac{3}{32\pi^2} \sum_i \int\limits_{q^2 > m^2} \frac{dq^2}{q^2} \left[m_{i\uparrow}^2(q^2) + m_{i\downarrow}^2(q^2) + \frac{1}{3} m_{il}^2(q^2) \right] , \tag{2}$$

where $\sum_i$ in (2) stands for the summation over all generations of up ($\uparrow$) and down ($\downarrow$) quarks and leptons (l). Correspondingly, the square of the W mass is equal

$$m_W^2 = g_2^2 \frac{3}{32\pi^2} \int\limits_{m_t^2}^{\infty} \frac{dq^2}{q^2} m_t^2(q^2) \; . \tag{3}$$

Here m_t is the top-quark mass, if we keep only the contribution of the heaviest quarks. The behaviour of $m_t^2(q^2)$ is well-known in the region where all couplings are small:

$$m_t^2(q^2) = m_t^2 \left[\frac{g_3(q^2)}{g_3(m_t^2)} \right]^{8/7} \left[\frac{g_1(m_t^2)}{g_1(q^2)} \right]^{1/10} \; . \tag{4}$$

According to (3), $m_t^2(q^2)$ changes with q^2 very slowly, and therefore, roughly,

$$m_W^2 \approx g_2^2 \frac{3}{32\pi^2} m_t^2 \ln \frac{\lambda_L^2}{m_t^2} \; . \tag{5}$$

In spite of the fantastically large value of the Landau scale, we have

$$\frac{1}{16\pi^2} \ln \frac{\lambda_L^2}{m_t^2} \sim 1 \tag{6}$$

and thus $m_W \sim m_t$. Taking into account the explicit dependence of $m_t^2(q^2)$ on q^2 according to (4), we find [3]

$$m_W^2 = \frac{3}{2} \frac{g_2^2(m_t^2)}{g_3^2(m_t^2)} \left\{ 1 - \left[\frac{g_3^2(\lambda_L^2)}{g_3^2(m_t^2)} \right]^{1/7} \right\} \; . \tag{7}$$

The accuracy of this result depends on the unknown contribution of the region $q^2 \geq \lambda_L^2$ and on the strong interaction in the region $q^2 \leq m_t^2$ as well as on the effective Yukawa couplings which appear in the theory due to non-zero quark masses.

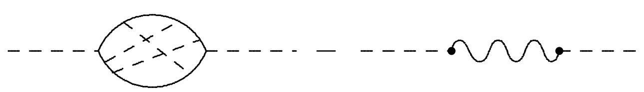
Because of the smallness of $\frac{1}{16\pi^2}$, which was compensated by the large value of $\ln \frac{\lambda_L^2}{m_t^2}$, the corrections coming from the region $q_t^2 \leq m_t^2$ are small. Corrections from the region $q^2 \geq \lambda_L^2$ will also be small provided $m_t^2(q^2)$ starts to decrease

faster (not logarithmically) at $q^2 > \lambda_L^2$. In the work which was done together with Yu. Dokshitzer [4] we have estimated the possible value of m_t from the known value of m_W by taking into account contributions of the Yukawa couplings, reasonable cut-off values in the region $q^2 > \lambda^2$ and different values for $g_3(m_t)$. We obtained the interval

$$180 \text{ GeV} < m_t < 200 \text{ GeV} \tag{8}$$

for m_t. If fermionic masses were created at a scale smaller than the Landau scale, the resulting top quark mass would, respectively, be larger.

Written in the form (2), the polarization operator $\Pi_{\mu\nu}$ for W is not transverse. In order to make it transverse, we have to add to the expression (1) the Goldstone contribution:

$\Pi_{\mu\nu} =$  (9)

Strictly speaking, it is necessary to have three Goldstone states in (9), and in this sense these three massless states are responsible for the masses of W and Z. At short distances, where the masses of fermions are small, the existence of three massless bosons implies, that there has to be a fourth massless state corresponding to a scalar boson, which, however, could become massive due to long distance interactions; this state is called the Higgs boson. In terms of the mass of the heaviest quark (the top quark) the mass squared of the Higgs boson is equal [3]

$$m_H^2 = 4\,\frac{\int m_t^4\,\frac{dq^2}{q^2}}{\int m_t^2\,\frac{dq^2}{q^2}}\ . \tag{10}$$

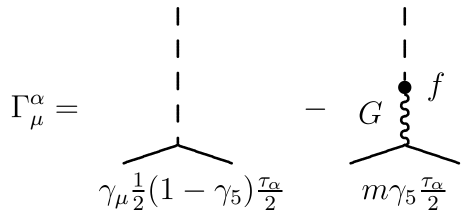
In the approximation which leads to the restriction (8), expression (10) gives the Higgs mass in the same range

$$180 \text{ GeV} < m_H < 200 \text{ GeV}\ . \tag{11}$$

3 The π meson mass

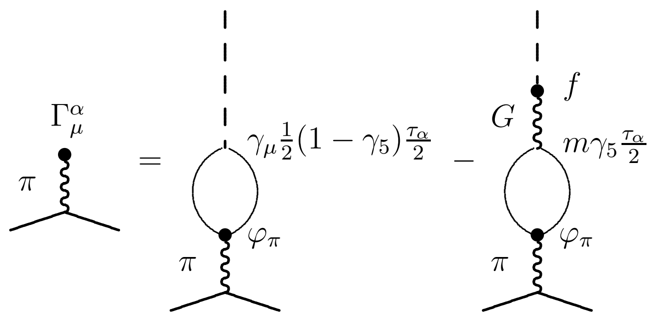
In order to see how the mass of the π meson could be influenced by the interaction which is responsible for the generation of fermionic masses, we can

start with an $SU(2)$ current. Due to the Goldstone contribution



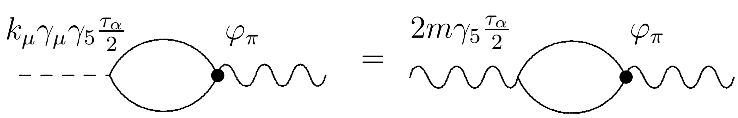
(12)

to this current (where m is the quark mass), it has to be conserved for any fermionic states, and for the light quark states in particular. For simplicity, we will neglect the mass difference of the u and d quarks. If we include the $SU(3)$ colour interaction and suppose, that this interaction produces a bound state called π, equation (12) will take the form



(13)

The current conservation $k_\mu \Gamma_\mu = 0$ implies that on the pion mass shell we have



(14)

In the first order in light quark masses φ_π can be calculated in the limit of zero quark masses; in this case φ_π is connected with the quark Green's function in a simple way. It is easy to show, that, if the latter is written in the form

$$G^{-1}(q) = Z\,(\hat{m} - \hat{q})\ , \tag{15}$$

then at large q^2 we have, in general:

$$\hat{m}(q) = m(q) + \frac{\nu^3(q)}{q^2}\ , \tag{16}$$

where m and ν^3 are slowly changing functions of q^2; the zero quark mass limit means $m = 0$. In this case π is the Goldstone and its coupling to fermions (i.e. its wave function) is

$$\varphi_\pi = \frac{Z}{f}\,\frac{\nu^3}{q^2}\,\gamma_5\ . \tag{17}$$

Combined with the expressions (15)-(17) the equation (14) leads to

$$k^2 f_\pi^2 = \frac{12}{16\pi^2}\int \frac{dq^2}{q^2} m\nu^3\ , \tag{18}$$

where the integration goes over the region $q^2 > \nu^2$ or $q^2 > \lambda^2_{QCD}$ for which (16) is correct; k^2 is the square of the pion mass and $f_\pi \approx 93$ MeV. The right-hand-side of (18) has the same structure as the expression (3) for the W mass: for slowly varying m and ν values it is proportional to $\ln\frac{\lambda_L^2}{\lambda^2_{QCD}}$ and this large ln factor is compensated by $16\pi^2$. The form (18) shows explicitly, how the pion mass feels the scale where the quark masses are created. One can show, that as a function of the strong coupling g_3 the product $m\nu^3$ behaves in the following way:

$$m(q)\nu^3(q) = m_0\nu_0^3\left(\frac{g_3(q)}{g_{30}}\right)^{8/21}\ ; \tag{19}$$

here m_0, ν_0 and g_{30} are the values of the quantities m, ν, g_3 at the minimal q^2 where (16) is correct. ($q^2 \approx \nu_0^2$). Using (18) and (19), we will find m_π^2 in the form

$$m_\pi^2 = \frac{12 m_0\nu_0^3}{16\pi^2 f_\pi^2}\ln\frac{\lambda_L^2}{\lambda_0^2}\ \left(\frac{g_3(\lambda_L^2)}{g_{30}}\right)^{8/21}\ . \tag{20}$$

Since we do not know ν, the value of m_π cannot be checked yet. However, being the part of the quark Green's function, it has to enter another physical quantity, like the ρ meson mass and therefore it can be measured. Calculating ν_0 by making use of (20) and a reasonable value for g_{30}^2, we find

$$230 \text{ MeV} < \nu < 260 \text{ MeV} \tag{21}$$

and $1/2 < \alpha_3(\nu) < 1$. This value of ν is in excellent qualitative agreement with experiment, because, according to (15), (16), ν is the mass of the constituent quark ($G^{-1}(\hat{q} = \nu) = 0$ if $m_0 \ll \nu$). The formula (18) for m_π^2 is very close to the widely used expression for the π meson mass

$$m_\pi^2 = \frac{2\,m_q}{f_\pi^2} < \overline{\Psi}\Psi >\ . \tag{22}$$

Indeed, (22) can be obtained from (18), if we forget about Z in (15) and about the running of m.

References

[1] Landau L.D., Abrikosov A.A., Khalatnikov I.M., Dokl. AN SSSR 95 (1954) 773, Dokl. AN SSSR 95 (1954) 1177

[2] Gross D.J., Wilczek F., Phys. Rev. D8 (1973) 3633, Phys. Rev. D9 (1974) 980; Politzer H.D., Phys. Rev. Lett. 30 (1973) 1346

[3] Gribov V.N., Phys. Lett. B336 (1994) 243

[4] Dokshitzer Yu.L., Gribov V.N., unpublished